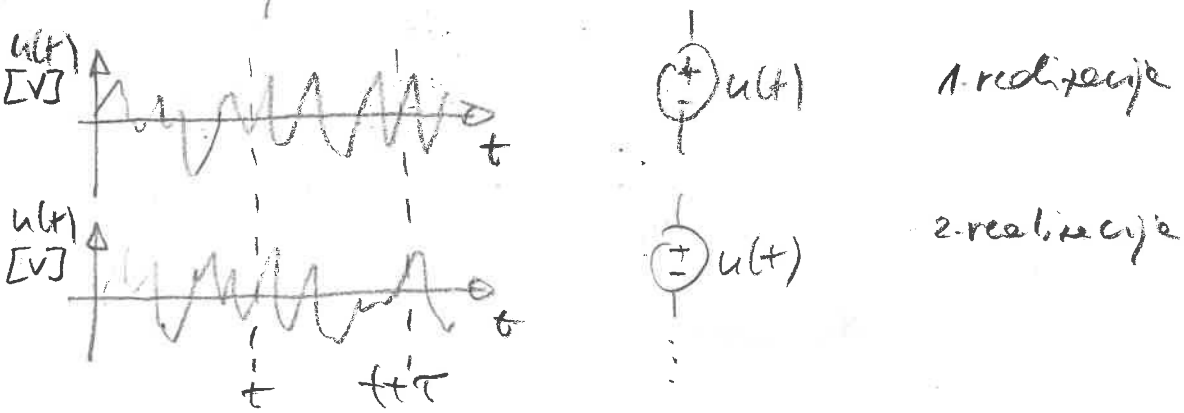
Karakterizacija šuma



Avtokorelacijska funkcija

$$C_{uu}(t,\tau) = E\left[u(t)\cdot u(t+\tau)\right]$$

t .. čas
τ .. razmik

↳ povprečje preko vseh realizacij

Stacionaren signal ... lastnosti niso odvisne od časa (t)

$$C_{uu}(t,\tau) = C_{uu}(\tau) = E\left[u(t)\cdot u(t+\tau)\right]$$

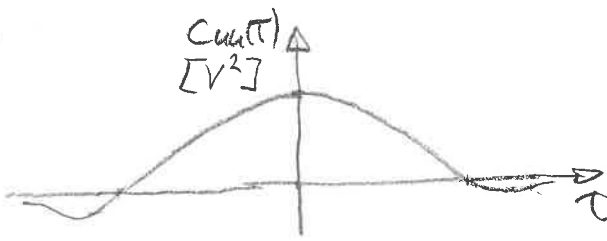
povprečje ni odvisno od časa t, samo razmika τ

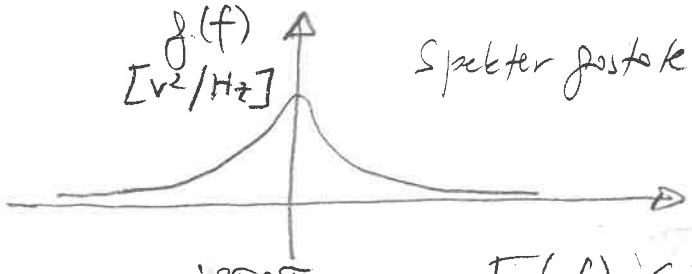
Ergodičen signal: povprečje preko vzorcev (realizacij) = časovno povprečje preko časa $(-\infty < t < \infty)$

$$C_{uu}(\tau) = E\left[u(t)u(t+\tau)\right] = \lim_{T\to\infty} \frac{1}{2T}\int_{-T} u(t)\cdot u(t+\tau)\,dt$$

Lastnosti $C_{uu}(\tau)$

1) $C_{uu}(0) \geq |C_{uu}(\tau)|$ maksimum pri $\tau = 0$, $C_{uu}(0) \geq 0$
2) $C_{uu}(\tau) = C_{uu}(-\tau)$ soda funkcija
3) $C_{uu}(0) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T} u^2(t)\,dt = R\Pi S(u(t))^2$ efektivna vrednost na kvadrat





Fourierova transforma $\mathcal{F}(C_{uu}(\tau)) = \int_{-\infty}^{\infty} C_{uu}(\tau)\,e^{-j2\pi f\tau}\,d\tau = F(f) \in \mathbb{R}$

↳ ker je $C_{uu}(\tau)$ soda in realna

$F(-f) = F(f)$ ... soda funkcija
(ker je $C_{uu}(\tau)$ soda funkcija)

$F(\omega) = \mathcal{F}(f(t))$

$f(t) \in \mathbb{R} \rightarrow F(\omega) = F(-\omega)^*$

$f(t) = f(-t) \rightarrow F(\omega) = F(-\omega)$

$f(t) = f(-t) \in \mathbb{R} \rightarrow F(\omega) = F(-\omega) \in \mathbb{R}$

Ker je $c_{uu}(\tau)$ avtokorelacijska funkcija, je $\mathcal{F}(c_{uu}(\tau)) = F(f) \geq 0$

Parsevalov teorem

$$c_{uu}(0) = RMS(u(t))^2 = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} u^2(t)\,dt = \int_{-\infty}^{\infty} F(f)\,df$$

Enostranski spektra gostote moči

$$S_{uu}(f) = 2F(f) = 2\int_{-\infty}^{\infty} c_{uu}(\tau)\, e^{-j2\pi f\tau}\,d\tau \in \mathbb{R}$$

definiran samo za $f \geq 0$

Lastnosti $S_{uu}(f)$

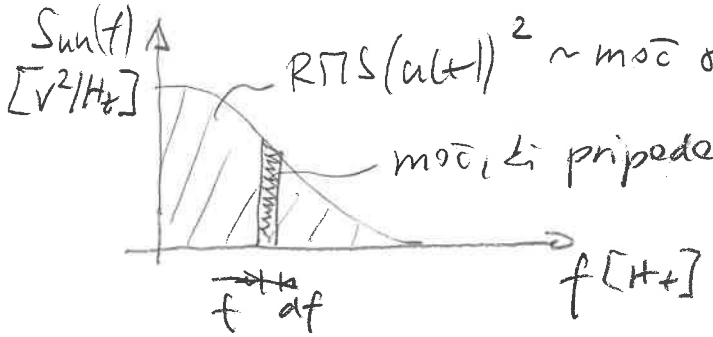
1) $S_{uu}(f) \geqslant 0$

2) Parsevalov teorem

$$\int_0^{\infty} S_{uu}(f)\,df = c_{uu}(0) = RMS(u(t))^2 = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} u^2(t)\,dt$$

$\Downarrow$

moč signala = kvadrat ef. vrednosti



$$c_{uu}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} u(t)\,u(t+\tau)\,dt = u(\tau) * u(-\tau)$$

$$\downarrow \qquad\qquad \downarrow$$

$$V(\omega) \qquad V(\omega)^*$$

$$\mathcal{F}(c_{uu}(\tau)) = V(\omega)V(\omega)^* = |V(\omega)|^2 \geq 0$$

$$c_{uu}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(f)\, e^{j\omega\tau}\,d\omega = \int_{-\infty}^{\infty} F(f)\, e^{j2\pi f\tau}\,\underbrace{\frac{d\omega}{2\pi}}_{df}$$

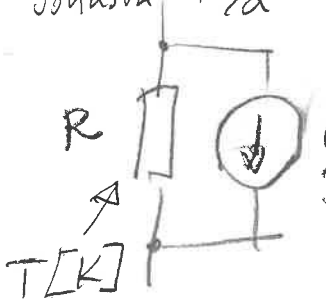
$$= \int_{-\infty}^{\infty} F(f)\, e^{j2\pi f\tau}\,df \quad \text{inverzní Fourier}$$

vstaví $\tau = 0$ → Parsevalův teorém

Vrste šuma v uporih

Termični šum ... naključno gibanje nosilcev naboja

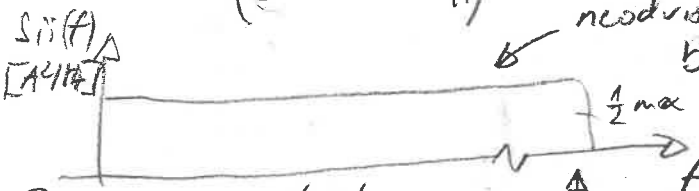
Johnson-Nyquistov šum



v uporih

$$S_{ii}(f) = \frac{4hf}{R\left(e^{\frac{hf}{kT}} - 1\right)} \approx \frac{4kT}{R} \quad \text{pri } 300K$$

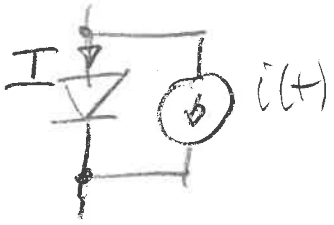
$f < 5\,THz$



h ... Planckova konstanta
k ... Boltzmannova konstanta
T ... temperatura v Kelvinih

Zrnati šum ... nosilci naboja gredo čez oviro (P–N spoj)

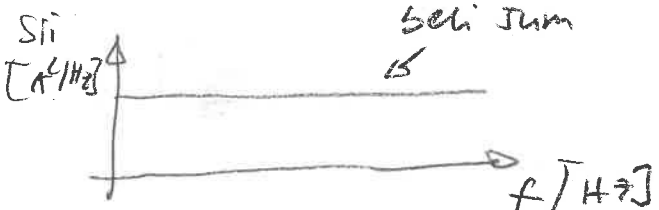
Shot noise



$$S_{ii}(f) = 2q\,I$$

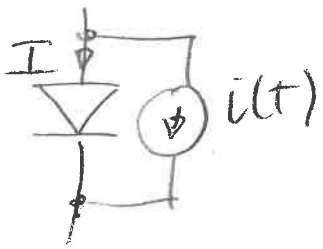
q ... naboj elektrona

v prevodnikih (kovinah) ga ni



1/f šum

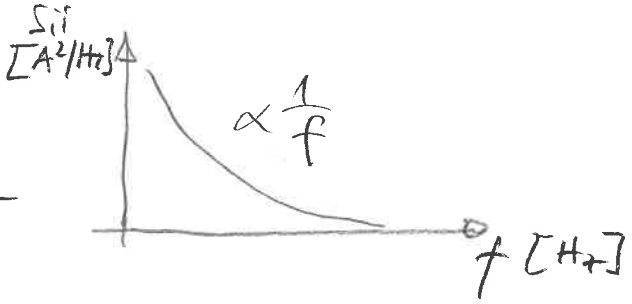
flicker noise



$$S_{ii}(f) = \frac{K_f I^{A_f}}{f}$$

dioda, bip.t., MOSFET

ogljenoplastni upori



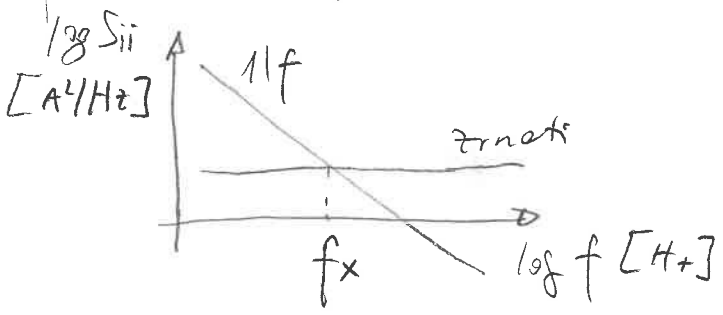
1/f šum = zrnati šum

$$2qI = \frac{K_f I^{A_f}}{f_x} \rightarrow f_x = \frac{K_f I^{A_f - 1}}{2q}$$

$I = 1\,mA$ $A_f = 1$

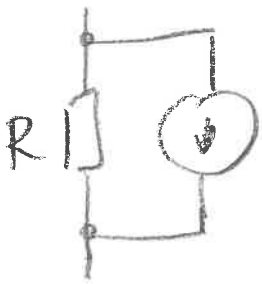
$K_f = 29 \cdot 10^{-18}\,A$

$f_x = 62\,Hz$



Šumni modeli elementov (za male signale)

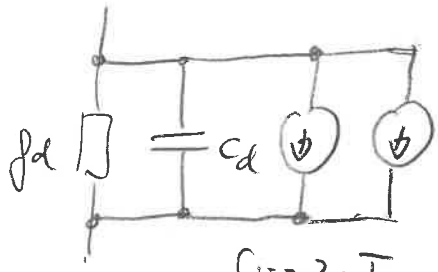
Upor



termični šum

$$S_{ii} = \frac{4kT}{R}$$

Dioda



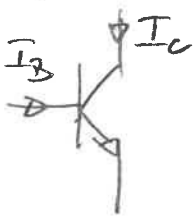
1/f šum

$$S_{ii} = \frac{K_f I_A^f}{f}$$

$S_{ii} = 2qI$

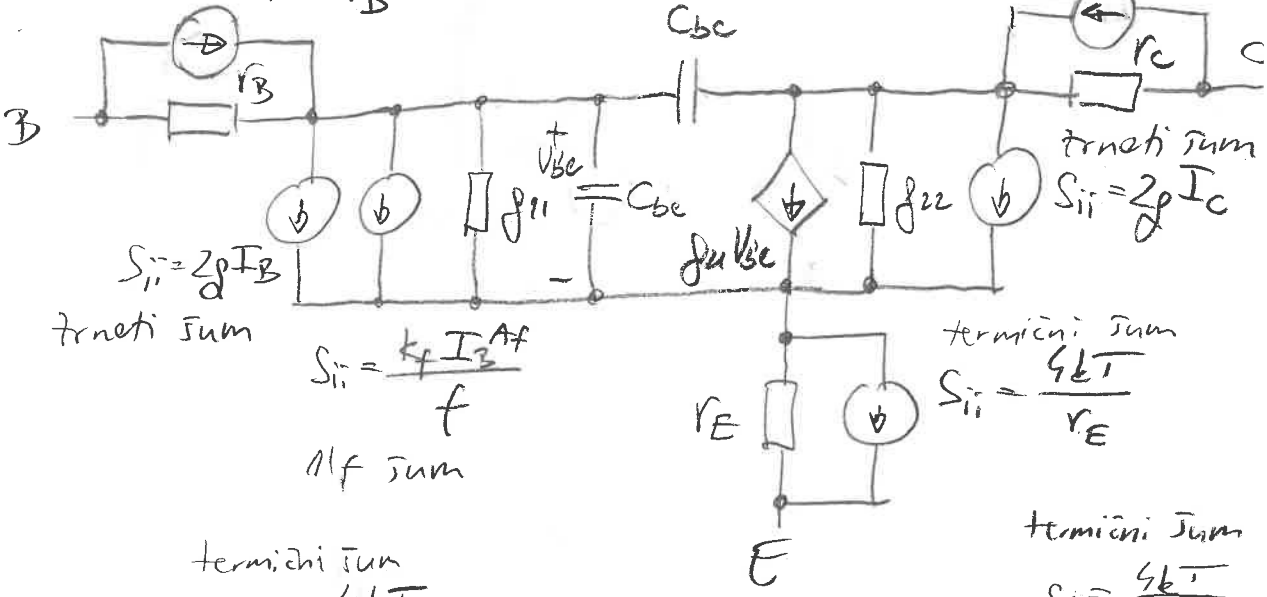
tvorni šum

Bipolarni tranzistor



termični šum

$$S_{ii} = \frac{4kT}{r_B}$$



termični šum

$$S_{ii} = \frac{4kT}{r_C}$$

tvorni šum

$S_{ii} = 2qI_C$

$S_{ii} = 2qI_B$

tvorni šum

$$S_{ii} = \frac{K_f I_B^{Af}}{f}$$

1/f šum

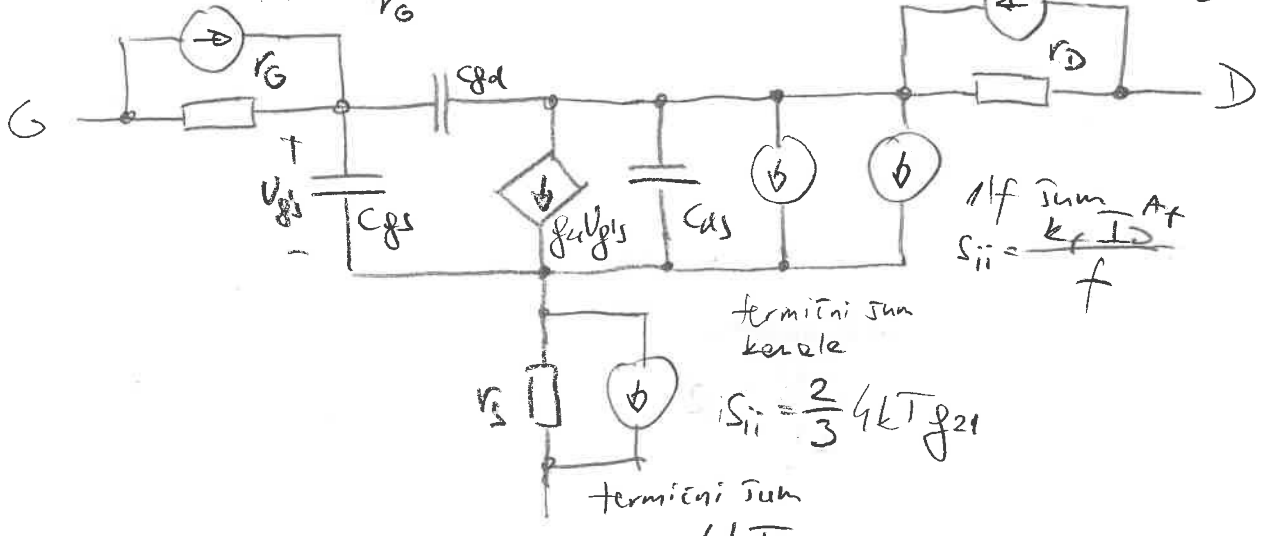
termični šum

$$S_{ii} = \frac{4kT}{r_E}$$

MOSFET

termični šum

$$S_{ii} = \frac{4kT}{r_G}$$



termični šum

$$S_{ii} = \frac{4kT}{r_D}$$

1/f šum

$$S_{ii} = \frac{K_f I_D^{Af}}{f}$$

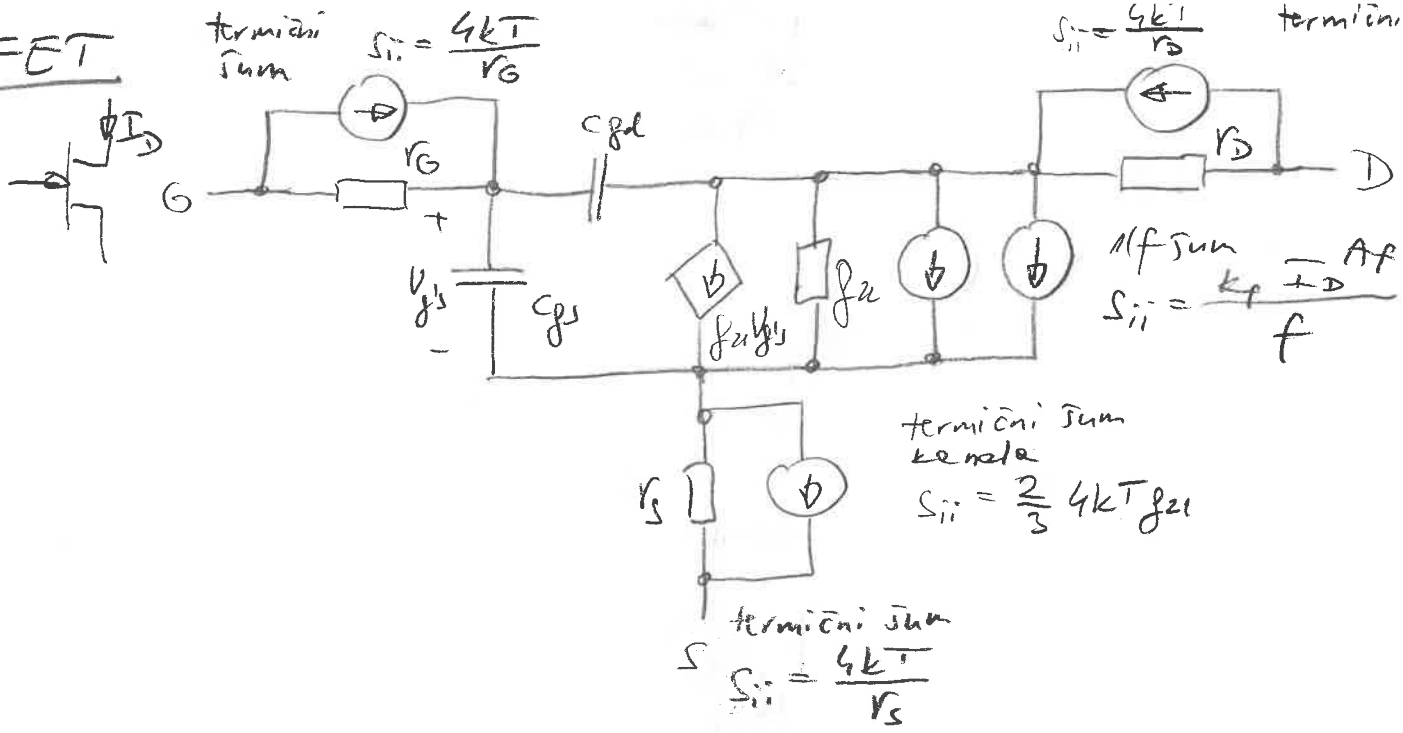
termični šum kanala

$$S_{ii} = \frac{2}{3} 4kT g_{21}$$

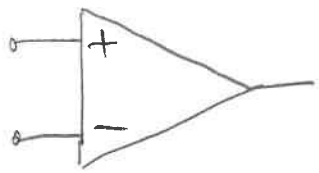
termični šum

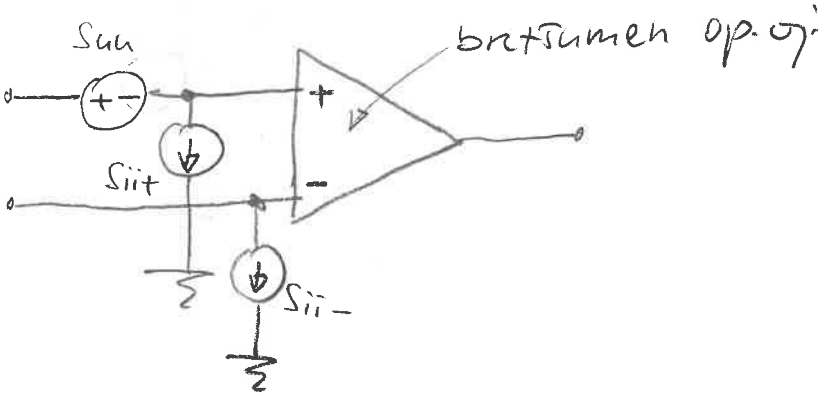
$$S_{ii} = \frac{4kT}{r_S}$$

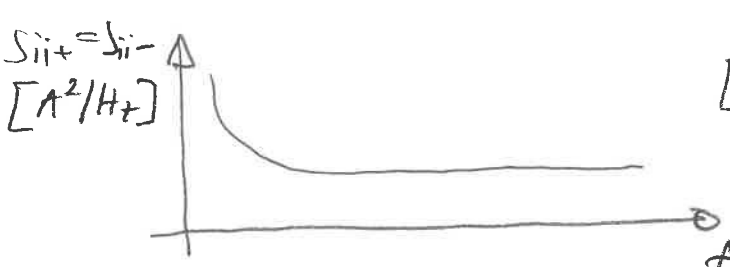
JFET

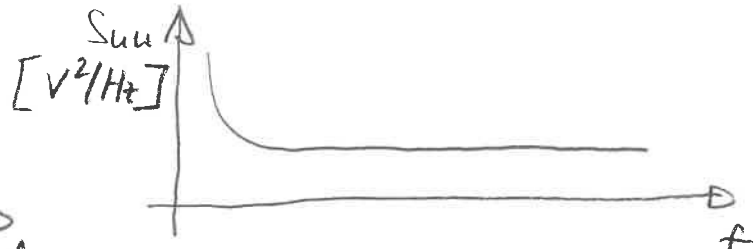


Operacijski ojačevalnik



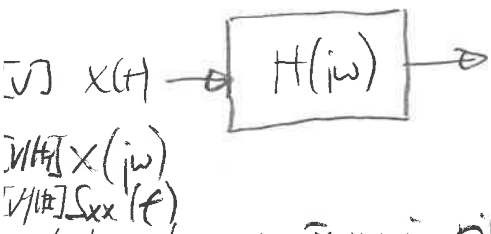






Včasih je spekter podan kot $\sqrt{S_{ii}}$ oz $\sqrt{S_{uu}}$ ($A/\sqrt{Hz}$ oz $V/\sqrt{Hz}$)

Prenos šuma skozi linearne vezje



determinističnih (sinusnih) signalov

$$Y(j\omega) = H(j\omega)\,X(j\omega)$$

$$S_{yy}(f) = |H(j\omega)|^2 S_{xx}(f)$$

šumnih signalov

Nekorelirani šumni signali

$$x(t) + y(t) = z(t)$$

šum šum šum

$S_{xx}(f)$ $S_{yy}(f)$ $S_{zz}(f)$

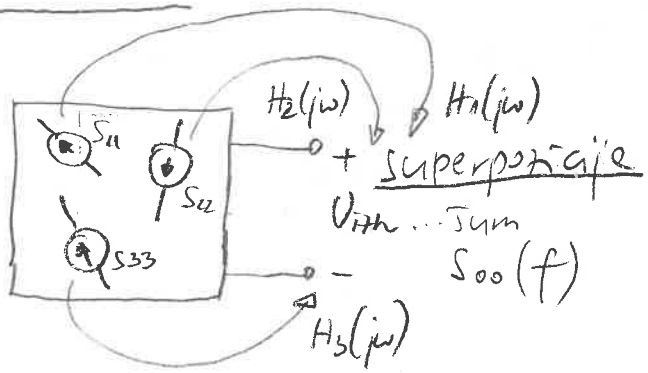
če ste $x(t)$ in $y(t)$ nekorelirana, potem

$$S_{zz}(f) = S_{xx}(f) + S_{yy}(f)$$

$x(t) \to \alpha x(t)$ .... $S_{xx}(f) \to \alpha^2 S_{xx}(f)$

Različni šumni viri znotraj enega elementa ter šumni viri dveh elementov so viri med seboj nekoreliranih.

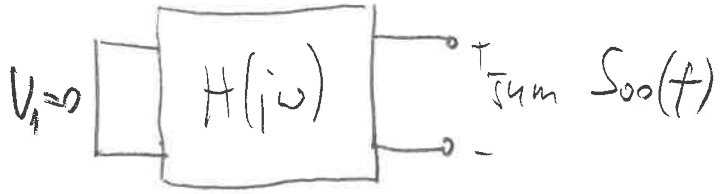
Izhodni šum

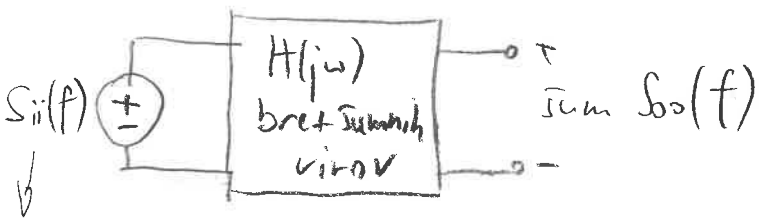


$$S_{00}(f) = |H_1(j2\pi f)|^2 S_{11}(f) + |H_2(j2\pi f)|^2 S_{22}(f) + |H_3(j2\pi f)|^2 S_{33}(f)$$

Ekvivalentni vhodni šum

$$V_2 = H(j\omega)\,V_1$$

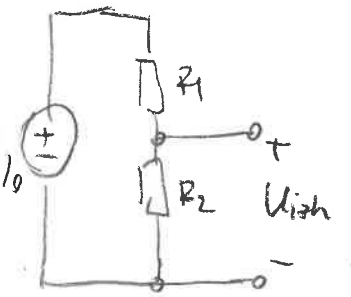




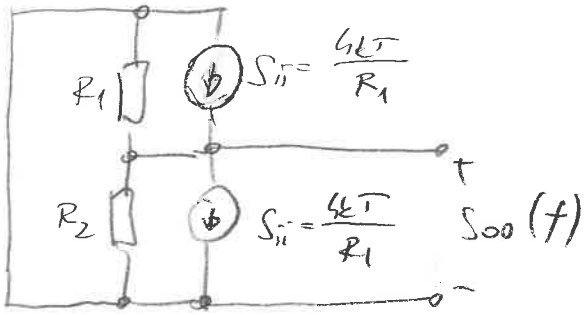
ekvivalenten vhodni šum

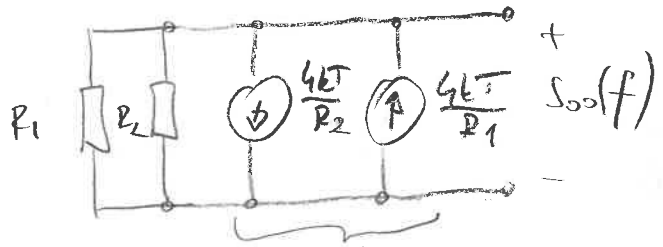
$$S_{ii}(f) = \frac{S_{00}(f)}{|H(j2\pi f)|^2}$$

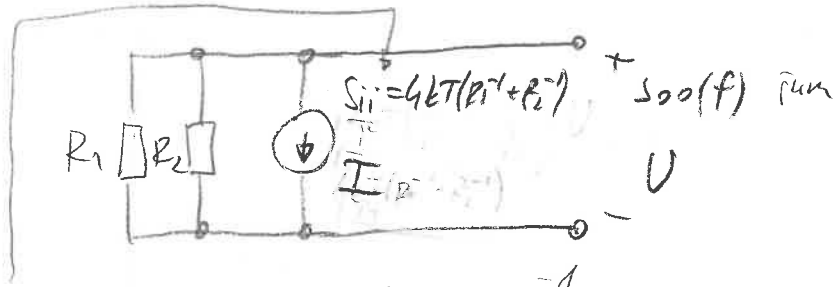
Primer: $T = 300K$, $V_0 = 6V$



Uizh šum
Sum šume
vstopni signal/šum
$B = 1kHz$ (instrument)
$R_1 = 10k$
$R_2 = 20k$





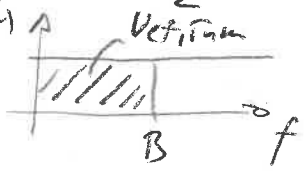


$X(t) + Y(t)$ ali $X(t) - Y(t) \rightarrow S_{xx}(f) + S_{yy}(f)$ $\quad S_{ii1} + S_{ii2}$ ne glede na smer

$$H(j\omega) = \frac{U}{I} = -\frac{1}{R_1^{-1} + R_2^{-1}}$$

$$S_{oo}(f) = |H(j\omega)|^2 \left(4kT(R_1^{-1} + R_2^{-1})\right) = \frac{1}{(R_1^{-1} + R_2^{-1})^2} \cdot 4kT(R_1^{-1} + R_2^{-1}) = \frac{4kT}{R_1^{-1} + R_2^{-1}}$$

$S_{oo}(f) = 1{,}10 \cdot 10^{-17}\ V^2/Hz$

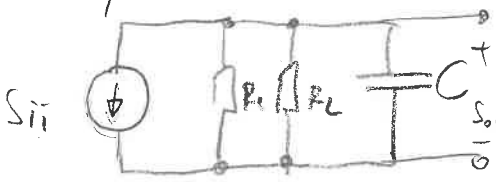


$V_{ef,šum} = \sqrt{S_{oo}(f) \cdot B} = 0{,}33\,\mu V$

$V_{ef,signal} = \frac{R_2}{R_1 + R_2} \cdot V_0 = \frac{4V}{\sqrt{2}}$

$$\frac{S}{N} = \frac{V_{ef,signal}}{V_{ef,šum}} = \frac{4}{\sqrt{2} \cdot 0{,}33 \cdot 10^{-6}} = 8{,}6 \cdot 10^6\ (139\ dB)$$

$C = 1\mu F$ na izhod



$$H(j\omega) = -\frac{1}{R_1^{-1} + R_2^{-1} + j\omega C} \qquad |H(j\omega)|^2 = \frac{1}{(R_1^{-1} + R_2^{-1})^2 + (\omega C)^2}$$

$$S_{oo}(f) = \frac{1}{(R_1^{-1} + R_2^{-1}) + (\omega C)^2} \cdot 4kT\underbrace{(R_1^{-1} + R_2^{-1})}_{G} = \frac{4kTG}{G^2 + \omega C^2}$$

$$V_{ef,šum}^2 = \int_0^\infty S_{oo}(f)\,df = 4kTG \int_0^\infty \frac{df}{G^2 + (2\pi fC)^2} = 4kTG \cdot \frac{\operatorname{arctg}\left(\frac{2\pi fC}{G}\right)}{2CG\pi}\Bigg|_0^\infty$$

$$= \frac{2kT}{C\pi} \cdot \left(\frac{\pi}{2} - 0\right) = \frac{kT}{C} = 4{,}14 \cdot 10^{-15}\ V^2$$

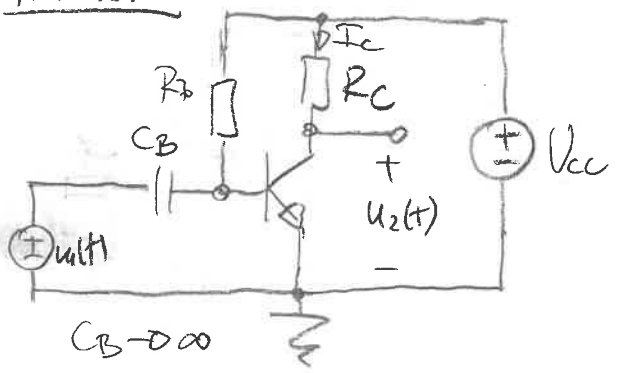
$V_{ef} = 64nV$ pri $R = \infty$

$\frac{S}{N} = \frac{4V}{\sqrt{2} \cdot 64nV} = 44 \cdot 10^6$ (153 dB)

pri $C = 1pF$ (10^6 x manjši C)

$\frac{S}{N} = 93\,dB$

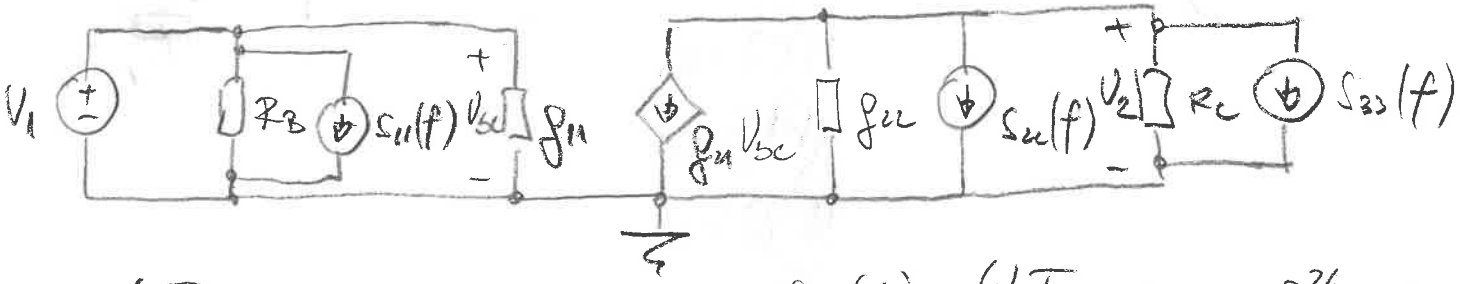
Primer:



$I_C = 1\,mA$
$R_B = 1M\Omega$
$R_C = 5k\Omega$
$g_{21} = 40\,mS$
$g_{22} = 10\,\mu S$
$g_{11} = 0{,}2\,mS$
$T = 300\,K$

Termični šum R_C, R_B
šumni šum tranzistorja

izhodni šum ($u_2(t)$)?
ekvivalentni vhodni šum?
srednje frekvence



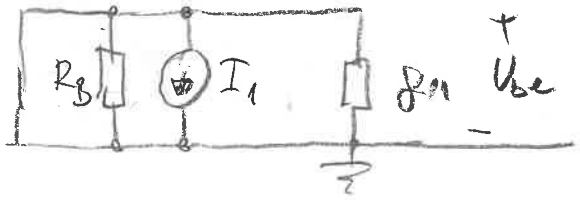
$$S_{II}(f) = \frac{4kT}{R_B} = 1{,}66 \cdot 10^{-26}\ A^2/Hz$$

$$S_{33}(f) = \frac{4kT}{R_C} = 3{,}31 \cdot 10^{-24}\ A^2/Hz$$

$$S_{22}(f) = 2qI_C = 3{,}20 \cdot 10^{-22}\ A^2/Hz$$

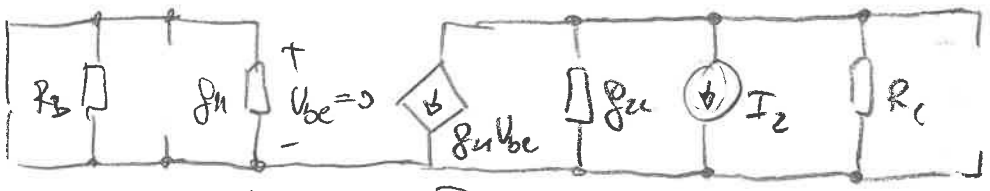
→ beli šum (frekvenčno neodvisen)

Izklopimo signalni vir ($V_1 = 0$)



$V_{be} = 0$ (tok vira I_1 se kratko sklene)

$V_2 = 0 \rightarrow H_1(j\omega) = \frac{V_2}{I_1} = 0 \quad [V/A]$

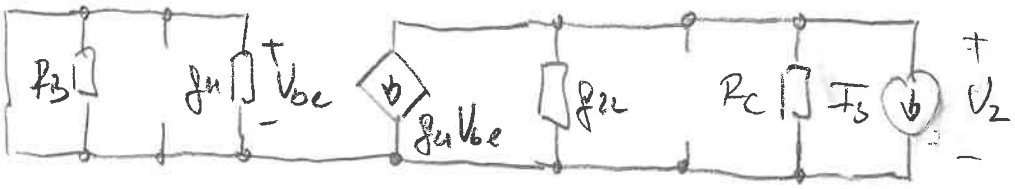


$g_m V_{be} = 0$

I_2 teče čez $R_C \| g_{22}^{-1}$

$= \frac{1}{g_{22} + R_C^{-1}}$

$$V_2 = -(R_C \| g_{22}^{-1})\, I_2 = -\frac{I_2}{g_{22} + R_C^{-1}} \qquad H_2(j\omega) = \frac{V_2}{I_2} = -\frac{1}{g_{22} + R_C^{-1}} \quad [V/A]$$



$$H_3(j\omega) = \frac{V_2}{I_3} = H_2(j\omega) = -\frac{1}{g_{22} + R_C^{-1}}$$

izhodni šum: $S_{OO}(f) = |H_1(j\omega)|^2 S_{II} + |H_2(j\omega)|^2 S_{22} + |H_3(j\omega)|^2 S_{33}$

$$= \frac{1}{(g_{22} + R^{-1})^2}\left(2qI_C + \frac{4kT}{R_C}\right) = 7{,}34 \cdot 10^{-15}\ V^2/Hz$$

$2qI_C$: $3{,}20 \cdot 10^{-22}$ A^2/Hz — $\frac{4kT}{R_C}$: $3{,}31 \cdot 10^{-24}$ A^2/Hz

V^2/A^2

Če izhod pomerimo + voltmetrom, ki meri efektivno vrednost in ima pasovno širino $B = 1\,kHz$, dobimo

$$V_{ef} = \sqrt{S_{oo}(f)^2 \cdot B} = 2{,}71\,\mu V$$

S 100x večjo pasovno širino dobimo 10x več: $V_{ef} = 27{,}1\,\mu V$

Prenosna funkcija od vhoda proti izhodu

$$H(j\omega) = \frac{V_2}{V_1} = -\frac{g_m}{g_{22} + \frac{1}{R_C}} = -190{,}5$$

Ekvivalentni vhodni šum

$$S_{in}(f) = \frac{S_{oo}(f)}{|H(j\omega)|^2} = 2{,}02 \cdot 10^{-19}\ V^2/Hz$$