A four-element lumped-parameter equivalent circuit, consisting of a resistance, an inductance, and two capacitances, has been found to represent the feed-point impedance of a dipole antenna. The values of these elements are related only to the physical dimensions of the antenna, not the frequency of operation. Empirical formulas are given for all the elements. The equivalent circuit gives negligible errors in radiation resistance and reactance for dipole half-lengths less than 0.1A, rising to 1% for resistance and 6% for reactance at 0.25A. It can be readily used in standard computer software packages such as SPICE, PSPIE, and MICROCAP.

I. INTRODUCTION

It is customary to represent the feed-point impedance of an electrically small dipole antenna by a lumped-element equivalent circuit consisting of a capacitance in series with a small frequency-dependent resistance (radiation resistance). This simple model has two distinct disadvantages: 1) the equivalent circuit becomes grossly dependent resistance (radiation resistance). This simple model has two distinct disadvantages: 1) the equivalent circuit becomes grossly dependent on the frequency of operation. Empirical formulas are given for all the elements. The equivalent circuit gives negligible errors in radiation resistance and reactance for dipole half-lengths less than 0.1A, rising to 1% for resistance and 6% for reactance at 0.25A. It can be readily used in standard computer software packages such as SPICE, PSpICE, and MICROCAP.

II. EQUIVALENT CIRCUITS DERIVED FROM

Chu [2] and Streable and Pearson [3] in practice, a dipole antenna is usually operated at or near its first resonant frequency. However, electrically small antennas, such as active receiving antennas, operate well below the resonant frequency. Thus, for these cases, higher order modes of resonance may be ignored. Two simpler equivalent circuits may then be deduced from [2] and [3]. These are, respectively

1. a three-element equivalent circuit from [2];
2. a four-element equivalent circuit from [3].

For a monopole antenna with an infinite groundplane, the values of L and R are halved while C is doubled.

A. The Three-Element Equivalent Circuit [2]

When higher order modes are ignored, Chu's equivalent circuit is reduced to that shown in Fig. 1. The values of \( L_{11}, C_{11}, \) and \( R_{11} \) will depend on the physical dimensions of the dipole antenna, independent of the frequency of operation. It is to be noted that \( R_{11} \) itself is not the radiation resistance. Their values are determined as follows.

The antenna feed-point impedance \( Z_0 = R_0 + jX_0 \) may be obtained from various sources or calculated using one of the classical antenna analysis methods. The value of \( C_{11} \) is determined from the reactance of the antenna at a frequency \( f \) much lower than the resonant frequency. In this process, it is assumed that \( L_{11} \) and \( R_{11} \) may be ignored, and \( C_{11} \) is given by

\[
C_{11} = \frac{|X_0|}{2\pi f} \tag{1a}
\]

Alternatively, the value of \( C_{11} \) is related to the dipole half-length \( h \) and radius \( a \) by [4]

\[
C_{11} = \frac{27.82 \times 10^{-12} h}{\ln (2h/a) - 1.693} \tag{1b}
\]

The inductance \( L_{11} \) and resistance \( R_{11} \) are estimated at the resonant frequency \( \omega_0 \) at which the reactance of the antenna vanishes and the radiation resistance is \( R_{ao} \)

\[
L_{11} = \frac{1}{\omega_0^2 C_{11}} + C_{11} R_{ao} \tag{1c}
\]

\[
R_{11} = \frac{L_{11}}{C_{11} R_{ao}} \tag{1d}
\]

It is found that in practice this equivalent circuit is reasonably accurate for the reactive component of the antenna impedance when the

Fig. 1. Three-element equivalent circuit of a dipole [2] \((h = 0.9 \, m, \quad a = 0.00264 \, m; \quad h/a = 341)\).
antenna is electrically short, but grossly inaccurate for the resistive component. It tends to predict a higher radiation resistance below resonance.

B. The Four-Element Equivalent Circuit [3]

Similarly to the three-element equivalent circuit, a four-element equivalent circuit results if higher order modes are ignored in Streible and Pearson's proposal. This is shown in Fig. 2. The values of \( R_{21}, R_{22}, C_{21}, \) and \( L_{21} \) are given by [3]

\[
\begin{align*}
R_{21} &= 0.663 \Omega \\
R_{22} &= 2200.6 \Omega \\
C_{21} &= \frac{0.00272}{\pi c} \text{F} \\
L_{21} &= \frac{434.55}{\pi c} \text{H}
\end{align*}
\]

where \( c = 3 \times 10^8 \text{ (m/s)} \); and \( l = 2h \) = total length of dipole (m). It is noted that in the formulas above, the radius of the dipole antenna has not been included. By comparing the values of \( C_{11} \) [(1b)] and \( C_{21} \) [(2c)], it is found that \( (2c) \) is valid only for wires with \( h/a = 341 \).

The input impedance for a dipole with half-length \( h = 0.9 \text{ m} \) calculated using \( (2a)-(2d) \) shows large errors when compared with the input impedance calculated using the King-Middleton quasi-zero approximation. The greatest error occurs when the frequency of operation is well below the resonant frequency and the radiation resistance is less than approximately \( 2 \Omega \). This is due to the presence of \( R_{21} \), which sets the minimum value of the series radiation resistance.

III. NEW FOUR-ELEMENT EQUIVALENT CIRCUIT

After an extensive empirical examination of many different \( L, C, \) and \( R \) combinations, the equivalent circuit of Fig. 3 was found to track the dipole impedance very closely. It consists of four elements and is of similar configuration to the three-element circuit of Chu [2], except for the added capacitance \( C_{32} \).

The values of the elements in Fig. 3 are obtained by “point matching” the resulting impedance against the theoretical values from the induced emf method. The resulting impedance of the proposed equivalent circuit coincides with the theoretical values at four points, comprising two on the radiation resistance and two on the antenna reactance curves. As an example, when the theoretical values from Wolff’s equations [6, equations (3.120) and (3.174)] are used, the empirical equations for the four elements are as follows:

\[
\begin{align*}
C_{11} &= \frac{12.0674h}{\log (2h/a) - 0.7245} \text{pF} \\
C_{32} &= 2h \left( \frac{0.89075}{\log (2h/a)} - 0.861 - 0.02541 \right) \text{pF} \\
L_{31} &= 0.2h \left[ (1.4813 \log (2h/a))^{0.6} - 0.6188 \right] \text{H} \\
R_{31} &= 0.41288 \left[ \log (2h/a) \right]^2 + 7.10754(2h/a)^{-0.2389} - 7.27408k\Omega
\end{align*}
\]

These empirical equations take into account the physical dimensions of the dipole, viz., half-length \( h \) and radius \( a \), which are expressed in meters.

Fig. 4(a) and 4(b) shows the percentage errors of the antenna feed-point impedance as a function of \( h/a \) ratio for dipole half-lengths of 0.1, 0.2, 0.25, and 0.3 wavelength (\( \lambda \)). It can be seen that for
TABLE I
COMPARISON OF ANTENNA INPUT IMPEDANCES AGAINST WOLFF [6] (h = 0.9 m; a = 0.00264 m; h/a = 341)

<table>
<thead>
<tr>
<th>Normalized Length 2h/λ</th>
<th>Frequency (MHz)</th>
<th>Input Impedance (ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2h/λ</td>
<td>f</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.125</td>
<td>0.125</td>
<td>20.83</td>
</tr>
<tr>
<td>0.250</td>
<td>0.250</td>
<td>41.67</td>
</tr>
<tr>
<td>0.375</td>
<td>0.375</td>
<td>62.50</td>
</tr>
<tr>
<td>0.500</td>
<td>0.500</td>
<td>83.33</td>
</tr>
</tbody>
</table>

Table I shows the typical antenna input impedances for the equivalent circuits of Figs. 1–3 for comparison purposes. It is obvious from the table that the values from Fig. 3 and Wolff [6] agree very well. Fig. 5 compares the three types of equivalent circuit graphically and numerically.
clearly demonstrates the superiority of the proposed new four-element equivalent circuit.

Table II compares the antenna input impedances of the new equivalent circuit against values given by Elliott [7, Table 7.5]. The general qualitative agreement is good.

IV. CONCLUSION

The feed-point impedance of a dipole or monopole antenna can be represented by a lumped-element equivalent circuit that has $L$, $C$, and $R$ elements whose values are only related to the physical dimensions of the antenna, independent of the frequency of operation. A proposed four-element equivalent circuit is shown to have an error of less than 1% for the radiation resistance and 6% for the reactance, for operation up to the dipole’s natural resonant frequency. Also, it is adequate to represent the impedance for dipole half-lengths up to approximately 0.3 wavelength.

The equivalent circuit can be readily used in standard computer software packages such as SPICE, PSpice, and MicroCap to analyze circuits containing dipole or monopole antennas with passive and active devices. It is particularly useful for analysis and design of electrically small nonresonant antenna elements such as used in active antennas and untuned frequency hopping antennas.

REFERENCES


Use of Equivalent Boundary Conditions for the Solution of a Class of Strip Grating Structures

Ronald R. DeLyser

Abstract—Equivalent boundary conditions for an electromagnetic strip grating are used to derive reflection and transmission coefficients for a boundary value problem consisting of three nonconducting media where the strip grating exists at one of the planar boundaries. The results are used to generate data for a number of cases where data and/or theories exist. The theory presented here compares favorably with the literature cited for the limiting condition of the period of the grating small when compared with the free-space wavelength of the radiation.

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Fig. 1 shows the boundary value problem of three dielectric regions of which regions 1 and 2 are separated by a strip grating of zero thickness, period $p$, gap width $a$, and strip width $b$. The incident and reflected waves in each region are at the angles shown relative to the normals of the interface planes. The boundary conditions at the two boundaries are that the tangential electric field intensity and the normal magnetic flux density are continuous. The tangential magnetic field intensity is continuous at the right boundary. At the left boundary the equivalent boundary conditions for the strips, which are oriented along the $z$ axis, are

$$E_z = \frac{l_2}{2} [j\omega\mu_0 (H_z - H_0) + \frac{1}{\epsilon_0} \frac{\partial}{\partial z} (\epsilon_2 E_z - \epsilon_1 E_z) ] \tag{1}$$

$$H_z - H_{z1} = 2l_1 \left( -j\omega\epsilon_0 E_y + \frac{1}{\mu_0} \frac{\partial B_z}{\partial z} \right), \tag{2}$$

with

$$l_1 = \frac{p}{\pi} \ln \sec \frac{\pi b}{2p} = \frac{p}{\pi} \ln \csc \frac{\pi a}{2p} \tag{3}$$

$$l_2 = \frac{p}{\pi} \ln \csc \frac{\pi b}{2p} = \frac{p}{\pi} \ln \sec \frac{\pi a}{2p} \tag{4}$$

The development of equivalent boundary conditions for periodic surfaces is not new to the literature. Since this communication deals with the applications of these boundary conditions, reference is made to some of that literature [1]-[4] and to a short history of their development given in [5].

The use of equivalent boundary conditions for a periodic grating of ribbon conductors for solving boundary value problems is, to the author’s knowledge, limited to cases where transmission and reflection coefficients [1] are derived for normal incidence on a grid in free space; where a computational study of strip grating antennas [6] was reported and closed-form analytical expressions were derived from which to calculate the resonance frequencies for certain cases of those antennas; and where equivalent boundary conditions [7] were used to predict surface waves along a metal grating on a grounded dielectric slab. It is the purpose of this paper to show that their use for a class of problems where the period of the grating is small compared with the free-space wavelength of the incident radiation is valid and fairly simple to implement.

II. SOLUTION PROCEDURE

The feed-point impedances of the new equivalent circuit against values given by Elliott [7, Table 7.5]. The general qualitative agreement is good.